

Whistler Wave Cascades in Solar Wind Plasma

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ABSTRACT

Nonlinear three dimensional, time dependent, fluid simulations of whistler wave turbulence are performed to investigate role of whistler waves in solar wind plasma turbulence in which characteristic turbulent fluctuations are characterized typically by the frequency and length scales that are respectively bigger than ion gyro frequency and smaller than ion gyro radius. The electron inertial length is an intrinsic length scale in whistler wave turbulence that distinguishably divides the high frequency solar wind turbulent spectra into scales smaller and bigger than the electron inertial length. Our simulations find that the dispersive whistler modes evolve entirely differently in the two regimes. While the dispersive whistler wave effects are stronger in the large scale regime, they do not influence the spectral cascades which are describable by a Kolmogorov-like $k^{-7/3}$ spectrum. By contrast, the small scale turbulent fluctuations exhibit a Navier-Stokes like evolution where characteristic turbulent eddies exhibit a typical $k^{-5/3}$ hydrodynamic turbulent spectrum. By virtue of equipartition between the wave velocity and magnetic fields, we quantify the role of whistler waves in the solar wind plasma fluctuations.

Key words: (magnetohydrodynamics) MHD, (Sun:) solar wind, Sun: magnetic fields, ISM: magnetic fields

1 INTRODUCTION

The solar wind is an excellent in-situ laboratory for investigating nonlinear and turbulent processes in a magnetized plasma fluid since it comprises a multitude of spatial and temporal length-scales associated with an admixture of waves, fluctuations, structures and nonlinear turbulent interactions. The in-situ spacecraft measurements (Matthaeus & Brown 1988, Goldstein et al 1995) reveal that the solar wind fluctuations, extending over several orders of magnitude in frequency and wavenumber, describe the power spectral density (PSD) spectrum that can be divided into three distinct regions (Goldstein et al 1995, Leamon et al 1999). The frequencies, for instance, smaller than 10-5 Hz lead to a PSD that has a spectral slope of -1. This follows the region that extends from 10-5 Hz to or less than ion/proton gyrofrequency where the spectral slope exhibits an index of -3/2 or -5/3. The latter, a somewhat controversial issue, is characterized essentially by fully developed turbulence and can be followed from the usual magnetohydrodynamic (MHD) description. The spacecraft observations have further revealed that length scales beyond the MHD regime, that are smaller than ion gyro radius ($k\rho_i \gg 1$) and temporal scales bigger

than ion cyclotron frequency $\omega > \omega_{ci} = eB_0/m_e c$, (where $k, \rho_i, \omega_{ci}, e, B_0, m_e, c$ are respectively characteristic mode, ion gyroradius, ion cyclotron frequency, electronic charge, mean magnetic field, mass of electron and speed of light) exhibit a spectral break where the inertial range slope of the solar wind turbulent fluctuations varies between -2 and -5 (Smith et al 1990, Goldstein et al 1994, Leamon et al 1999). Notably, the dynamics of the length-scales in this region cannot be described by the usual MHD models that possess characteristic frequencies smaller than ion gyro frequencies. At 1 AU, the ion inertial length scales are smaller than ion gyro radii in the solar wind (Goldstein et al 1995). The latter, associated with plasma motion due to finite Larmor radii, can readily be resolved in the usual MHD models by introducing Hall terms to accommodate the ion gyro scales up to scales as small as ion inertial length scales. The higher time resolution databases identifying the spectral break indicate that Alfvénic MHD cascades (Smith et al 1990, Goldstein et al 1994, Leamon et al 1999) are terminated near the spectral break. The characteristic modes in this region are observed to evolve typically on timescales involving the dispersive kinetic Alfvénic fluctuations. The onset of the second or the kinetic Alfvén inertial range is still elusive to our understanding of the solar wind turbulence and many other nonlinear interactions. Specifically, the mechanism leading to the spectral break has been thought to be either

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mediated by the kinetic Alfvén waves (KAWs) (Hasegawa 1976), or by electromagnetic ion-cyclotron-Alfvén (EMICA) waves (Gary et al 2008; Wu & Yoon 2007), or by a class of fluctuations that can be dealt within the framework of the HMHD plasma model (Alexandrova et al 2007, 2008; Shaikh & Shukla 2008, 2008a). Stawicki et al (2005) argue that Alfvén fluctuations are suppressed by proton cyclotron damping at intermediate wavenumbers so the observed power spectra are likely to consist of weakly damped magnetosonic and/or whistler waves which are dispersive unlike Alfvén waves. Moreover, turbulent fluctuations corresponding to the high frequency and $k\rho_i \gg 1$ regime lead to a decoupling of electron motion from that of ion such that the latter becomes unmagnetized and can be treated as an immobile neutralizing background fluid. While whistler waves typically survive in the higher frequency (and the corresponding smaller length scales) part of the solar wind plasma spectrum, their role in influencing the inertial range turbulent spectral cascades is still debated (Biskamp et al 1996; Shaikh & Zank 2003; Shaikh & Zank 2005; Shaikh & Shukla 2009; Shaikh & Shukla 2008). The Kolmogorov like dimensional arguments indicate that propagation of whistlers in the presence of a mean or an external constant magnetic field may change the spectral index of the inertial range turbulent fluctuations from $k^{-7/3}$ to k^{-2} (Biskamp et al 1996). By contrast, the numerical simulations (Biskamp et al 1996; Shaikh & Zank 2003; Shaikh & Zank 2005) suggest that whistler waves do not influence the spectral migration of turbulent energy in the inertial range despite strong wave activity and that the turbulent spectra corresponding to the electron fluid fluctuations in whistler wave turbulence continue to exhibit a Kolmogorov-like $k^{-7/3}$ spectrum. What is not clear from these work is the quantitative role of whistler and the corresponding mode coupling interactions that mediate the inertial range turbulent spectra. Furthermore, the whistler wave turbulence described in the Refs. (Biskamp et al 1996; Shaikh & Zank 2003; Shaikh & Zank 2005) focus on purely two dimensional interactions, ignoring thus the variations in the third dimension. It is therefore unclear whether the nonlinear whistler mode coupling interactions in three dimensions modify the energy cascades in the inertial range turbulence. Intrigued largely by these issues, the primary goal of this paper is to investigate the nonlinear interaction amongst whistler waves and turbulent fluctuations, based on nonlinear fluid simulations, in $\omega > \omega_{ci}$ regime where correlation length scales of turbulence are comparable to the electron inertial length scales. Understanding the role of whistler waves in the turbulent cascades is crucial to many other space plasma processes since whistler waves, in addition to solar wind turbulence, are instrumental in governing nonlinear processes in numerous other plasma systems that range from solar wind (Krafft & Volokitin 2003; Saito et al 2008; Stawicki et al 2005; Gary et al 2008; Ng et al 2003; Vocks et al 2005; Salem et al 2007; Bhattacharjee et al 1998), magnetic reconnection in the Earth's magnetosphere (Wei et al 2007) to interstellar medium (Burman 1975) and astrophysical plasmas (Roth 2007) where characteristic fluctuations can typically be of several astronomical units. These are only a few of the numerous other studies. For more literature, the

readers can refer to the simulation work by Biskamp (Biskamp et al 1996) and others including Shukla (1978), Shukla et al (2001), Cho & Lazarian (2004), Galtier (2008), Urrutia et al (2008), Saito et al (2008), Bengt & Shukla (2008), Shaikh (2009) and numerous references therein.

In this paper, I focus on understanding the nonlinear turbulent cascades mediated by whistler waves in a fully three dimensional geometry. Our objective is to investigate the role of whistlers in establishing the turbulent equipartition amongst the modes that are responsible for the nonlinear mode coupling interactions which critically determine the inertial range power spectra. Remarkably, we find that despite the equipartition processes mediated by whistler modes for which the wave activity is strong, the inertial range spectra continue to exhibit a Kolmogorov-like spectrum where whistler wave effects are *unimportant*. We begin in Section 2 by describing the underlying whistler wave turbulence model and its linear properties. Section 3 describes nonlinear simulation results of inertial range turbulent spectra. In section 4, we discuss the theoretical arguments corresponding to the whistler wave turbulent spectra that correspond to the characteristic length scales smaller as well as bigger than electron inertial length (d_e). The process of turbulent equipartition between the magnetic and velocity field, quantifying the whistler wave effects, is also described in this section. Finally, section 5 summarizes our results.

2 WHISTLER WAVE TURBULENCE MODEL

Whistler modes are excited in the solar wind plasma when the characteristic plasma fluctuations propagate along a mean or background magnetic field with frequency $\omega > \omega_{ci}$ and the length scales are $c/\omega_{pi} < \ell < c/\omega_{pe}$, where ω_{pi}, ω_{pe} are the plasma ion and electron frequencies. The electron dynamics plays a critical role in determining the nonlinear interactions while the ions merely provide a stationary neutralizing background against fast moving electrons and behave as scattering centers. The whistler wave turbulence can be described by the electron magnetohydrodynamics (EMHD) model of plasma (Kingsep et al. 1990) that deals with the single fluid description of quasi neutral plasma. The EMHD model has been discussed in considerable detail in earlier work (Kingsep et al. 1990; Biskamp et al 1996; Dastgeer et al. 2000a; Dastgeer et al. 2000b; Shaikh & Zank 2003; Shaikh & Zank 2005). In whistler modes, the currents carried by the electron fluid are important, and we therefore write down only those equations which are pertinent to electron motion. These are electron fluid momentum, electric field, currents, and electron continuity equations,

$$m_e n \frac{\partial \mathbf{V}_e}{\partial t} + \mathbf{V}_e \cdot \nabla \mathbf{V}_e = -en\mathbf{E} - \frac{ne}{c} \mathbf{V}_e \times \mathbf{B} - \nabla P - \mu m_e n \nabla \mathbf{V}_e, \quad (1)$$

$$\mathbf{E} = -\nabla \phi - \frac{1}{c} \frac{\partial \mathbf{A}}{\partial t}, \quad (2)$$

$$\nabla \times \mathbf{B} = \frac{4\pi}{c} \mathbf{J} + \frac{1}{c} \frac{\partial \mathbf{E}}{\partial t}, \quad (3)$$

$$\frac{\partial n}{\partial t} + \nabla \cdot (n \mathbf{V}_e) = 0. \quad (4)$$

The remaining equations are $\mathbf{B} = \nabla \times \mathbf{A}$, $\mathbf{J} = -en\mathbf{V}_e$, $\nabla \cdot \mathbf{B} = 0$. Here m_e, n, \mathbf{V}_e are the electron mass, density and

fluid velocity respectively. \mathbf{E}, \mathbf{B} respectively represent electric and magnetic fields and ϕ, \mathbf{A} are electrostatic and electromagnetic potentials. The remaining variables and constants are, the pressure P , the collisional dissipation μ , the current due to electrons flow \mathbf{J} , and the velocity of light c . The displacement current in Ampere's law Eq. (3) is ignored, and the density is considered as constant throughout the analysis. The electron continuity equation can therefore be represented by a divergence-less electron fluid velocity $\nabla \cdot \mathbf{V}_e = 0$. The electron fluid velocity can then be associated with the rotational magnetic field through

$$\mathbf{V}_e = -\frac{c}{4\pi ne} \nabla \times \mathbf{B}. \quad (5)$$

On taking the curl of Eq. (1) and, after slight rearrangement of the terms, we obtain

$$\frac{\partial \mathbf{P}}{\partial t} - \mathbf{V}_e \times (\nabla \times \mathbf{P}) + \nabla \xi = -\mu m_e \mathbf{V}_e \quad (6)$$

where

$$\mathbf{P} = m_e \mathbf{V}_e - \frac{e\mathbf{A}}{c} \quad \text{and} \quad \xi = \frac{1}{2} m_e \mathbf{V}_e \cdot \mathbf{V}_e + \frac{P}{n} - e\phi.$$

Here \mathbf{P} is generalized electron momenta. The curl of Eq. (6) eliminates the gradient of the scalar quantity (the third term from the left in the lhs) and yields

$$\frac{\partial \boldsymbol{\Omega}}{\partial t} + \nabla \times (\mathbf{V}_e \times \boldsymbol{\Omega}) = -\mu m_e \nabla \times \mathbf{V}_e, \quad (7)$$

where

$$\boldsymbol{\Omega} = \nabla \times \mathbf{P} = d_e^2 \nabla^2 \mathbf{B} - \mathbf{B}.$$

It can be seen from Eq. (6) that in the ideal whistler mode turbulence (i.e. neglecting the term associated with the damping μ), the Curl of generalized electron momenta is frozen in the electron fluid velocity. This feature is strikingly similar to Alfvénic turbulence where the magnetic field is frozen in the ideal two fluid plasma (Biskamp 2003). On substituting $\boldsymbol{\Omega}$ into the above equation and using appropriate vector identities, we obtain the three-dimensional equation of EMHD describing the evolution of the magnetic field fluctuations in whistler wave,

$$\begin{aligned} \frac{\partial}{\partial t} (\mathbf{B} - d_e^2 \nabla^2 \mathbf{B}) + \mathbf{V}_e \cdot \nabla (\mathbf{B} - d_e^2 \nabla^2 \mathbf{B}) - \\ (\mathbf{B} - d_e^2 \nabla^2 \mathbf{B}) \cdot \nabla \mathbf{V}_e = \mu d_e^2 \nabla^2 \mathbf{B}. \end{aligned} \quad (8)$$

The length scales in Eq. (8) are normalized by the electron skin depth $d_e = c/\omega_{pe}$ i.e. the electron inertial length scale, the magnetic field by a typical amplitude B_0 , and time by the corresponding electron gyro-frequency. In Eq. (8), the diffusion operator on the right hand side is raised to $2n$. Here n is an integer and can take $n = 1, 2, 3, \dots$. The case $n = 1$ stands for normal diffusion, while $n = 2, 3, \dots$ corresponds to hyper- and other higher order diffusion terms.

The linearization of Eq. (8) about a constant magnetic field $\mathbf{B} = B_0 \hat{z} + \tilde{\mathbf{B}}$, where B_0 and $\tilde{\mathbf{B}}$ are respectively constant and wave magnetic fields, yields the following equation,

$$\omega_k (1 + d_e^2 k^2) \tilde{\mathbf{B}} + \frac{CB_0}{4\pi ne} i k_{\parallel} \mathbf{k} \times \tilde{\mathbf{B}} = 0. \quad (9)$$

On eliminating the wave perturbed magnetic field from the above relation, one obtains the following dispersion relation,

$$\omega_k = \omega_{c_0} \frac{d_e^2 k_y k}{1 + d_e^2 k^2}, \quad (10)$$

where $\omega_{c_0} = eB_0/mc$ and $k^2 = k_x^2 + k_y^2$. The use of Eq. (10) in Eq. (9) leads to the following relation between the wave magnetic field and the velocity field,

$$\tilde{\mathbf{B}} = \pm \frac{i}{k} \mathbf{k} \times \tilde{\mathbf{B}} \quad (11)$$

The rhs of Eq. (11), in combination with Eq. (5), corresponds essentially to the whistler wave perturbed velocity field. This equation indicates that whistler waves are transverse and are produced by rotational magnetic field that leads essentially to the velocity field fluctuations. On replacing the rhs in Eq. (11) with the perturbed velocity field, it can be shown that the whistler modes obey equipartition between the magnetic and velocity field components as $k^2 |\mathbf{B}|^2 \simeq |\mathbf{V}_e|^2$. The whistler wave activity can thus be quantified by how closely the characteristic modes obey the turbulent equipartition relation. In section 4, we will investigate using nonlinear 3D fluid simulations the equipartition mediated by whistler waves in the inertial range turbulent spectra to signify the role of whistlers in the solar wind plasma.

It becomes evident from Eq. (8) that there exists an intrinsic length scale d_e , the electron inertial skin depth, which divides the entire turbulent spectrum into two regions; namely short scale ($kd_e > 1$) and long scale ($kd_e < 1$) regimes. In the regime $kd_e < 1$, the linear frequency of whistlers is $\omega_k \sim k_y k$ and the waves are dispersive. Conversely, dispersion is weak in the other regime $kd_e > 1$ since $\omega_k \sim k_y/k$ and hence the whistler wave packets interact more like the eddies of hydrodynamical fluids. The equation of EMHD [Eq. (8)] is also exactly integrable, yielding the total energy integral;

$$E = \frac{1}{2} \int (1 + d_e^2 \nabla^2) |\mathbf{B}|^2 d^3 \mathbf{v},$$

and generalized helicity. Here $d^3 \mathbf{v}$ is a 3D volume element. In the presence of dissipation (μ) the total energy decays eventually with time since

$$\frac{\partial E}{\partial t} = -\mu d_e^2 \int d^3 \mathbf{v} [(\nabla^2 B)^2]. \quad (12)$$

Hence the inclusion of dissipation will damp the smaller scale fluctuations in the whistler wave turbulence. The damping of the smaller dissipative scales are not expected to influence the inertial range turbulent cascades.

3 SIMULATION RESULTS

Turbulent interactions mediated by the coupling of whistler waves and inertial range fluctuations are studied in three dimensions (3D) based on a nonlinear 3D whistler wave turbulence code that we have developed at Center for Space Plasma and Aeronomic Research (CSPAR), the University of Alabama in Huntsville (UAH). Our code numerically integrates Eq. (8). The spatial discretization employs a pseudospectral algorithm (Gottlieb et al 1977; Shaikh & Zank 2006; Shaikh & Zank 2007) based on a Fourier harmonic expansion of the bases for physical variables (i.e. the magnetic field, velocity), whereas the temporal integration uses a Runge Kutta (RK) 4th order method. The boundary conditions are periodic along the x, y and z directions in the local rectangular region of the solar wind plasma.

Figure 1. 3D simulation of whistler wave turbulence in the $kd_e < 1$ regime exhibits a Kolmogorov-like inertial range power spectrum close to $k^{-7/3}$. The simulation parameters are: Box size is $L_x \times L_y \times L_z = 2\pi \times 2\pi \times 2\pi$, numerical resolution is $N_x \times N_y \times N_z = 200 \times 200 \times 200$, electron skin depth is $d_e = 0.015$, magnitude of constant magnetic field is $B_0 = 0.5$. The characteristic large scales in this regime possess strong dispersion and wave activity that can be quantified from the turbulent equipartition between the velocity and magnetic fields.

The turbulent fluctuations are initialized by using a uniform isotropic random spectral distribution of Fourier modes concentrated in a smaller band of lower wavenumbers ($k < 0.1 k_{max}$). While spectral amplitudes of the fluctuations are random for each Fourier coefficient, it follows a certain initial spectral distribution proportional to $k^{-\alpha}$, where α is an initial spectral index. The spectral distribution set up in this manner initializes random scale turbulent fluctuations. We note that a constant magnetic field is included along the z direction (i.e. $\mathbf{B}_0 = B_0 \hat{\mathbf{z}}$) to accommodate the large scale (or the background solar wind) magnetic field. The size of the 3D computational domain is $(2\pi)^3$ with the spectral resolution 256^3 . In this paper, we present the results of freely decaying whistler wave turbulence and focus primarily on understanding the inertial range cascades in both the $kd_e < 1$ and $kd_e > 1$ regimes. In principle, turbulence can be driven. The driven whistler turbulence is nonetheless beyond the scope of this paper.

Electron whistler fluid fluctuations, in the presence of a constant background magnetic field, evolve by virtue of nonlinear interactions in which larger eddies transfer their energy to smaller ones through a forward cascade. According to (Kolmogorov 1941), the cascades of spectral energy

Figure 2. The small scales magnetic field fluctuations in the $kd_e > 1$ regime depicts a Kolmogorov-like $k^{-5/3}$ spectrum which is a characteristic of hydrodynamic fluid. The simulation parameters are same as those used in Fig 1, except $d_e = 0.15$. Our simulations show that the small scale fluctuations evolve as non magnetized eddies of hydrodynamic fluid where whistler waves do not influence the energy cascades.

occur purely amongst the neighboring Fourier modes (i.e. local interaction) until the energy in the smallest turbulent eddies is finally dissipated gradually due to the finite dissipation. This leads to a damping of small scale motions. By contrast, the large-scales and the inertial range turbulent fluctuations remain unaffected by direct dissipation of the smaller scales. Since there is no mechanism that drives turbulence at the larger scales in our model, the large-scale energy simply migrates towards the smaller scales by virtue of nonlinear cascades in the inertial range and is dissipated at the smallest turbulent length-scales. The spectral transfer of turbulent energy in the neighboring Fourier modes in whistler wave turbulence follows a Kolmogorov phenomenology (Kolmogorov 1941; Iroshnikov 1963; Kraichnan 1965) that leads to Kolmogorov-like energy spectra. We find from our 3D simulations that whistler wave turbulence in the $kd_e < 1$ and $kd_e > 1$ regimes exhibits respectively $k^{-7/3}$ (see Fig 1) and $k^{-5/3}$ (see Fig 2) spectra. The inertial range turbulent spectra obtained from our 3D simulations are further consistent with 2D work (Biskamp et al 1996; Dastgeer et al. 2000a; Dastgeer et al. 2000b). Interestingly, it is evident from the whistler wave dispersion relation that the wave effects dominate in the large scale, i.e. $kd_e < 1$, regime where the inertial range turbulent spectrum depicts a Kolmogorov-like $k^{-7/3}$ spectrum. On the other hand, tur-

bulent fluctuations in the smaller scale ($kd_e > 1$) regime behave like non magnetic eddies of hydrodynamic fluid and yield a $k^{-5/3}$ spectrum. The wave effect is weak, or negligibly small, in the latter. Hence the nonlinear cascades are determined essentially by the hydrodynamic like interactions. The observed whistler wave turbulence spectra in the $kd_e < 1$ and $kd_e > 1$ regimes (Figs 1 & 2) can be followed from the Kolmogorov-like arguments (Kolmogorov 1941; Iroshnikov 1963; Kraichnan 1965) that describe the inertial range spectral cascades. We elaborate on these arguments to explain our simulation results of Fig. (1) & (2) in the following section.

4 ENERGY SPECTRA IN WHISTLER WAVE TURBULENCE

The exact spectral indices corresponding to the whistler wave turbulent spectra, described by the ideal electron magnetohydrodynamic invariant, can be understood from the Kolmogorov's dimensional arguments (Kolmogorov 1941; Iroshnikov 1963; Kraichnan 1965). The electron skin depth, d_e , in EMHD turbulence intrinsically divides the entire Fourier spectrum into regions for which length scales are either larger or smaller than d_e . We derive the spectral indices for both regions of the turbulent spectrum.

In the underlying whistler wave model of magnetized plasma turbulence, the inertial range eddy velocity is characterized typically by $v_e \sim \nabla \times \mathbf{B}$. Thus the typical velocity of the magnetic field eddy B_ℓ with a scale size ℓ can be represented by $v_e \simeq B_\ell/\ell$. The eddy turn-over time is then given by

$$\tau \sim \frac{\ell}{v_e} \sim \frac{\ell^2}{B_\ell}.$$

This is the time scale that predominantly leads to the nonlinear spectral transfer of energy in fully developed whistler wave turbulence. While the inertial range nonlinear cascades are determined essentially by the eddy turn over or spectral transfer time scale, it is not clear whether the characteristic length scales bigger than d_e , where whistler wave propagation dominate, are influenced by whistler interaction time scales. We will comment on this issue in the following along with the inertial range spectra in both the $kd_e < 1$ and $kd_e > 1$ regimes.

4.1 $kd_e < 1$: Whistler wave regime

In the regime where characteristic length scales are bigger than the electron skin depth ($kd_e < 1$), the inertial range whistler turbulent energy is dominated by the large scale fluctuations. The total energy corresponding to the turbulent fluctuations in this regime is then given as,

$$E \sim |\mathbf{B}|^2 \sim B_\ell^2 \sim v_e^2 \ell^2.$$

The B_ℓ represent magnetic field associated with the magnetic field eddy of length ℓ . The second similarity follows from the assumption of an equipartition of energy in the magnetic and velocity field components of whistler waves. The process of equipartition originates from the correlation between the velocity and magnetic field fluctuations

Figure 3. Turbulent equipartition between the velocity and magnetic fields is observed in our 3D simulations. The equipartition is measured for the entire turbulent spectrum at each time step by the relation $E_{equi} \simeq \sum_k (|v_e|^2 - k^2 |B_k|^2)$. When the characteristic turbulent modes evolve towards equipartition, the relationship $|v_e(\mathbf{k}, t)|^2 \simeq k^2 |B(\mathbf{k}, t)|^2$ is obeyed. Consequently, $E_{equi} \rightarrow 10^{-7}$. This number is small enough to establish a nearly perfect equipartition between the velocity and magnetic field associated with the whistler waves.

$v_e \sim \mathbf{k} \times \mathbf{B}$, where $\mathbf{k} = k_x \hat{x} + k_y \hat{y} + k_z \hat{z}$ is a three dimensional wave vector. The latter is further consistent with the electron flow speed, i.e. Eq (6) in combination with the wave perturbed magnetic field Eq. (11), that is used to derive the dynamical equation of whistler wave turbulence, i.e. Eq. (8). This velocity-magnetic field correlation essentially produces the velocity field fluctuations that are normal to the magnetic field in a whistler wave packet. Consequently, the energy associated with the velocity and magnetic field for each characteristic turbulent mode evolves toward a relationship that satisfies $v_e^2 \simeq k^2 B^2$. To quantify our arguments, we follow the evolution of turbulent equipartition in our simulations by computing the following quantity,

$$E_{equi}(t) \simeq \sum_k (|v_e(k, t)|^2 - k^2 |B(k, t)|^2), \quad (13)$$

which should be close to zero for the inertial modes that exhibit nearly perfect equipartition. The summation in Eq. (13) is carried over all the modes (i.e. k 's) that constitute the inertial range spectrum. Our simulation results, following the evolution of Eq. (13), are shown in Fig 3. We find from our simulations that the inertial range turbulent fluctuations closely follow the equipartition that leads to a strong

wave activity in the $kd_e < 1$ regime. Despite the presence of the dispersive whistler waves in this regime, the inertial range spectrum continues to follow a Kolmogorov-like $k^{-7/3}$ spectrum. The equipartition in whistler wave turbulence has also been reported in our two dimensional work (Dastgeer et al. 2000a; Dastgeer et al. 2000b; Shaikh & Zank 2005). Interestingly, our 3D simulations, describing the equipartition between the velocity and magnetic field fluctuations, are consistent with the 2D counterpart. It thus appears that the turbulent equipartition is a robust feature of whistler waves that is preserved in both 2D and 3D nonlinear mode coupling interactions. The spectral cascades of inertial range turbulent energy is nonetheless determined by the energy cascade per unit nonlinear time as follows,

$$\varepsilon \simeq \frac{E}{\tau} \simeq \frac{B_\ell^3}{\ell^2}.$$

On assuming that the spectral energy cascade is local in the wavenumber space (Kolmogorov 1941; Kolmogorov 1941; Iroshnikov 1963), the energy spectrum per unit mode yields $E_k \simeq \varepsilon^\alpha k^\beta$. On substituting the energy and energy dissipation rates and equating the powers of B_ℓ and ℓ , we obtain $\alpha = 2/3$ and $\beta = -7/3$. This, in the $kd_e < 1$ regime, leads to the following expression for the energy spectrum

$$E_k \simeq \varepsilon^{2/3} k^{-7/3}.$$

It can be noted from the dispersion relation, Eq. (10), that the group velocity of whistler waves in the $kd_e < 1$ regime is $\omega/k_y \sim \partial\omega/\partial k_y \sim k$ and the dispersion is $\omega_k \sim \omega_0 k_y k$. Both the quantities are proportional to the characteristic wavenumber k . It is evident from these relations that the group velocity and dispersion of whistlers are predominant at the smaller length scales in the $kd_e < 1$ regime. Correspondingly, the scaling law for energy cascades is modified by the short scale spectrum of the whistler waves in the $kd_e < 1$ regime. To determine the effect of small scale whistler waves on the spectral transfer, we compute the energy transfer rates in the $kd_e < 1$ regime as follows.

The $kd_e < 1$ regime comprises the dispersive whistler waves whose interaction time can be estimated from $\tau_w \simeq \ell/v_g$, where v_g is the group velocity for the whistler modes. The group velocity of whistler waves in the $kd_e < 1$ regime is $v_g \sim \partial\omega/\partial k_y \sim k \sim \ell^{-1}$. The interaction time between two (or more) whistler wave packets thus yields $\tau_w \sim \ell^2$. The nonlinear energy cascade rates computed as above, i.e. $\varepsilon \sim E/\tau$, will be modified by the whistler interaction time as

$$\varepsilon_w \sim \left(\frac{E}{\tau}\right) \left(\frac{\tau_w}{\tau}\right) \sim \frac{B^4}{\ell^2}.$$

Here ε_w is the whistler modified energy transfer rates. On using the Kolmogorov phenomenology that the spectral transfer is local and depends only on the energy dissipation rates and modes (Kolmogorov 1941; Kolmogorov 1941; Iroshnikov 1963), the energy spectrum can be given by $E_k \sim \varepsilon^\alpha k^\beta$. Upon substituting the energy dissipation rates, we estimate the spectral energy as $E_k \sim \varepsilon^{1/2} k^{-2}$. The change in the inertial range spectral slope due to the whistler waves is referred to as *whistler effect* (Biskamp et al 1996; Dastgeer et al. 2000a; Dastgeer et al. 2000b; Shaikh & Zank 2005). By introducing the whistler time scale in deriving the energy cascade rates ε_w , it is noteworthy that the spectrum in the $kd_e < 1$

regime is modified by the presence of whistler waves and one might infer that the whistler waves modify the inertial range spectrum from $k^{-7/3}$ to a more flatter one, i.e. k^{-2} . Although the difference between the two spectra is small enough to be noticeable in the 3D simulations (generally because of poor spectral resolutions), the flattening of the spectrum is not observed in our simulations that persistently show that the whistler wave spectrum is close to $k^{-7/3}$. While the spectral resolution in our three dimensional simulations is not adequate enough to resolve the two distinct spectra, very high resolution simulations (upto 5120^2) in 2D (Shaikh & Zank 2005) suggest that the volume integrated energy spectra are not affected by the presence of the whistler waves and the inertial range turbulent fluctuations continue to exhibit a Kolmogorov-like $k^{-7/3}$ spectrum. The whistler effect in those simulations (Shaikh & Zank 2005) is observed to be influential only at the local region in the inertial range turbulent spectrum. This result is further consistent with that of MHD turbulence (Shebalin et al. 1983) where anisotropy in the spectral space mediated by the Alfvén waves (i.e. the Alfvén effect) is explained by virtue of local Fourier mode, while the volume integrated MHD spectrum exhibits a Kolmogorov-like (Kolmogorov 1941) $k^{-5/3}$ power law. The controversy (Iroshnikov 1963; Kraichnan 1965) with regard to the $k^{-5/3}$ or $k^{-3/2}$ MHD spectrum is not a subject of this paper and we will not discuss it any further. The reader can however refer to the book by Biskamp (Biskamp 2003) and the reference therein.

4.2 $kd_e > 1$: Hydrodynamic-like regime

The regime $kd_e > 1$ in whistler wave turbulence corresponds essentially to a hydrodynamic regime because the EMHD equation, Eq. (8), in this regime reduces to the Navier Stokes equation that describes the dynamics of non magnetized hydrodynamic flows. The energy spectrum in this regime is dominated by the shorter length-scale turbulent eddies that give rise to the characteristic spectrum of an incompressible hydrodynamic fluid. The group velocity of whistlers, in this regime, is small and hence it is expected that the effect of whistlers will not be present. For $kd_e > 1$, the first term in the energy can be neglected and thus

$$E \sim k^2 B^2 \sim \frac{B_\ell^2}{\ell^2}.$$

where we have used $k \sim 1/\ell$. The energy cascade rates per unit nonlinear transfer time, $\varepsilon \simeq E/\tau$, in the regime $kd_e > 1$ lead to $\varepsilon \simeq B_\ell^3/\ell^4$. On using the Kolmogorov's phenomenology of local spectral cascade, the energy spectrum of whistler turbulence in the $kd_e > 1$ regime can be obtained as

$$E_k \sim \varepsilon^{2/3} k^{-5/3},$$

in agreement with our simulations (see Fig 2). This spectrum is identical to that of energy in three dimensional incompressible Navier-Stokes turbulence and further confirms the hydrodynamic nature of the whistler wave turbulence for the small scale fluctuations in $kd_e > 1$ regime. While the longer scales ($kd_e < 1$ modes) possess stronger tendency of behaving like whistlers, the shorter scales ($kd_e > 1$ modes) act like unmagnetized hydrodynamical eddies where wave

effects are considerably weaker. Hence whistler wave turbulence in this regime exhibits the energy spectrum that is essentially identical to that of hydrodynamic fluid.

5 SUMMARY

Three dimensional simulations of turbulent cascades in solar wind plasma are carried out to quantify the role of whistler waves corresponding to the inertial range fluctuations that possess characteristic frequency bigger than the ion gyro frequency ($\omega > \omega_{ci}$) and length scales smaller than the ion gyro radius ($k\rho_i > 1$). In this regime, the solar wind plasma fluctuations comprise of unmagnetized ions, hence the entire dynamics is governed by the electron fluid motions. The rotational magnetic field fluctuations in the presence of a background magnetic field lead to propagation of dispersive whistler waves in which the wave magnetic and velocity fields are strongly correlated through the equipartition ($v_e^2 \simeq k^2 B^2$). The latter is employed in our simulations to quantify the role of whistler waves that are ubiquitously present in the inertial range in the high frequency ($\omega > \omega_{ci}$) solar wind plasma. Interestingly we find that despite strong wave activity in the inertial range, whistler waves do not influence the inertial range turbulent spectra. Consequently, the turbulent fluctuations in the inertial range are described by Kolmogorov-like phenomenology. Thus consistent with the Kolmogorov-like dimensional argument, we find that turbulent spectra in the $kd_e < 1$ and $kd_e > 1$ regimes are described respectively by $k^{-7/3}$ and $k^{-5/3}$. Our results are important particularly in understanding turbulent cascade corresponding to the high frequency ($\omega > \omega_{ci}$) solar wind plasma where characteristic fluctuations are comparable to the electron inertial skin depth.

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